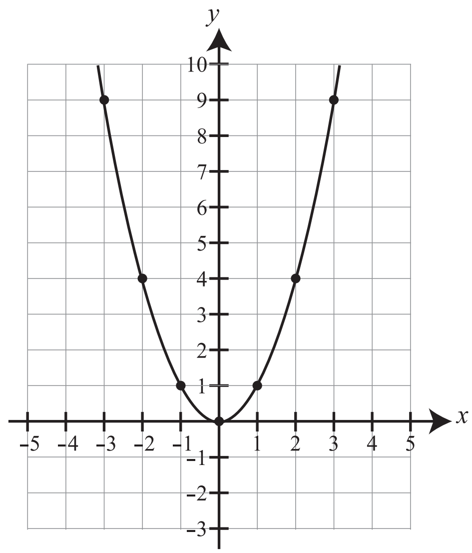
Section 3.5

The Quadratic Function $y = ax^2 + bx + c$

An equation of the form $y = ax^2 + bx + c$ with $a \neq 0$ is called a quadratic function and has a graph called a **parabola**.

The basic equation is $y = x^2$. (Here $a = 1$ and $b = c = 0$). The table below gives points on this graph as shown. They are connected with a smooth curve.

x	x^2	y
–3	$(-3)^2$	9
–2	$(-2)^2$	4
–1	$(-1)^2$	1
0	0^2	0
1	1^2	1
2	2^2	4
3	3^2	9



The point at the bottom of the graph is called the *vertex* of the parabola. The parabola is symmetric to the vertical line through the vertex. This line is called the *axis of symmetry*.

Facts about the graph of the parabola $y = ax^2 + bx + c$:

- The x-coordinate of the vertex is $-\frac{b}{2a}$. Substitute this number into the equation to find the y-coordinate of the vertex.
- The *axis of symmetry* is the vertical line $x = -\frac{b}{2a}$.
- The symmetry is apparent in the *table* as well. The y-values are the same for the x-values that are at the same distance from the vertex.
- If a is positive, the vertex is the minimum (lowest) point; the graph opens upward.
- If a is negative, the vertex is the maximum (top) point; the graph opens downward.
- If $|a| > 1$, the graph is narrower than the one above; if $0 < |a| < 1$, a proper fraction, the graph is wider than the parabola above.

Graphing parabolas

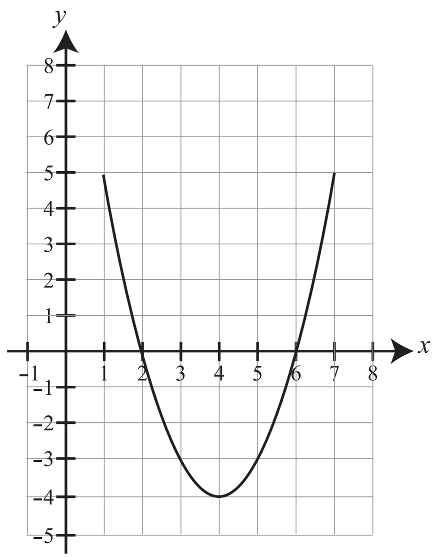
Method 1: If a particular set of values is given for x, then make a table for those values and plot the points. Draw a smooth curve in the shape of a parabola between them. Be sure to go all the way from one end to the other.

MODEL PROBLEM:

Graph $y = x^2 - 8x + 12$ for $1 \leq x \leq 7$.

Solution:

x	$x^2 - 8x + 12$	y
1	$1^2 - 8(1) + 12$	5
2	$2^2 - 8(2) + 12$	0
3	$3^2 - 8(3) + 12$	–3
4	$4^2 - 8(4) + 12$	–4
5	$5^2 - 8(5) + 12$	–3
6	$6^2 - 8(6) + 12$	0
7	$7^2 - 8(7) + 12$	5



Notice how the symmetry appears in the table as well as on the graph. The axis of symmetry is $x = -\frac{b}{2a} = -\frac{-8}{2(1)} = 4$. The vertex is $(4, -4)$. Once you have found the vertex and calculated the y-values on one side of the vertex, the other side's y-values are the same.

Method 2: If you are not given a domain for graphing, then follow the procedure in the model problem below.

MODEL PROBLEM:

Find the vertex and graph the parabola $y = -x^2 - 2x + 3$.

Solution: Note that $a = -1$, $b = -2$ and $c = 3$. Since a is negative, expect the graph to open downward.

First find the axes and vertex. The axis of symmetry is $x = -\frac{b}{2a} = -\frac{(-2)}{2(-1)} = -1$.

Substitute this x-value to find the y-value of the vertex: $-(-1)^2 - 2(-1) + 3 = 4$.

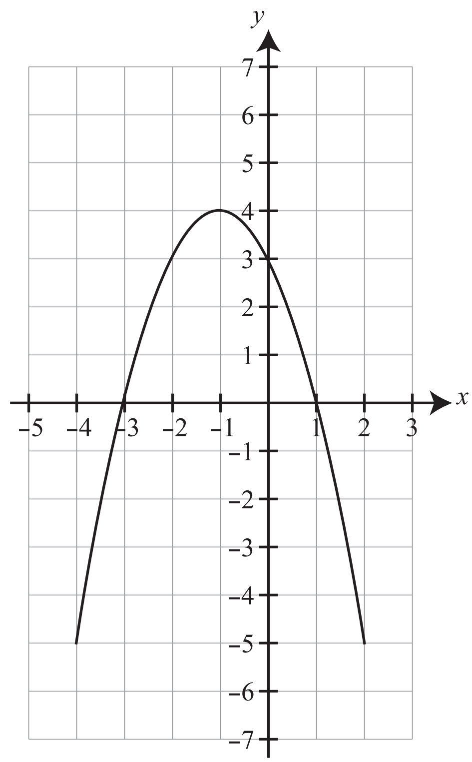
The vertex is $(-1, 4)$.

Next: Make a table on one side of the vertex. Use the symmetry to find points on the other side of the vertex.

x	$-x^2 - 2x + 3$	y
–4		–5
–3		0
–2		3
–1	Vertex	4
0	$-0^2 - 2(0) + 3$	3
1	$-1^2 - 2(1) + 3$	0
2	$-2^2 - 2(2) + 3$	–5

From the bottom half of table, by symmetry. (rows $x = -4, -3, -2$)

Finally: Draw the graph. As expected, this one opens downward.



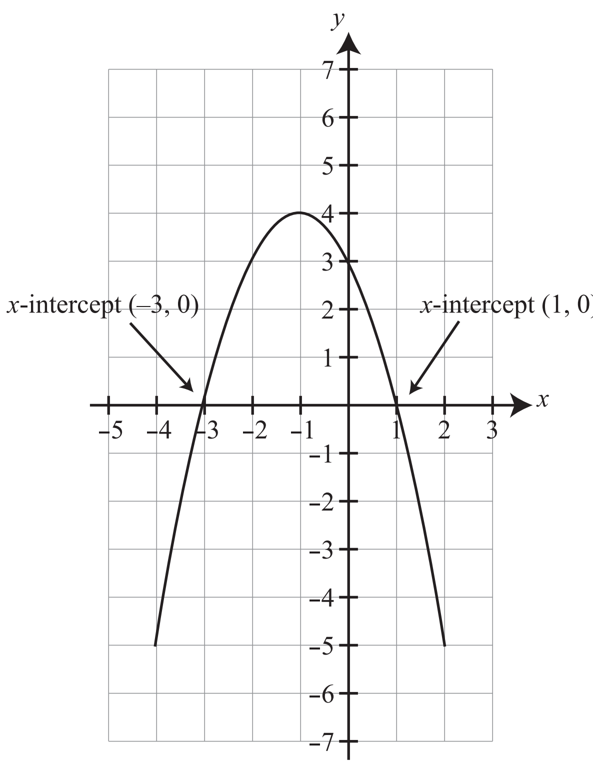
Graphing Solutions to Quadratic Equations

To solve an equation by graphing, including a quadratic equation, first graph the equation and then determine its *x*-intercepts. These values are where the *y*-coordinate, and therefore the equation, equals zero.

MODEL PROBLEM:

Solve $-x^2 - 2x + 3 = 0$ by graphing.

Solution: Graph $y = -x^2 - 2x + 3$ (See model problem above). The graph is



Answer: The *x*-intercepts are $(-3, 0)$ and $(1, 0)$, so the solutions are $x = -3$ or $x = 1$.